Ex. Reparametrize $\vec{r}(t) = \langle 3\sin(t), 2t, 3\cos(t) \rangle$ by arc length measured from $t=0$

Sol. 1) Compute arc length function

$$s(t) = \int_{q=0}^{t} |\vec{r}\,'(q)|\, dq$$

$$r'(q) = \langle 3\cos(t), 2, -3\sin(t) \rangle$$

$$|r'(q)| = \sqrt{3^2\cos^2(t) + 2^2 + 3^2\sin^2(t)}$$

$$= \sqrt{3^2 + 2^2}$$

$$|\vec{r}\,'(q)| = \sqrt{13}$$

$$s(t) = \int_{q=0}^{t} \sqrt{13}\, dq$$

$$s(t) = \sqrt{13}\,(t - 0)$$

$$s(t) = \sqrt{13}\, t$$

Time is determined by arc length

$$t = \frac{s}{\sqrt{13}}$$

derived from there

2) Replace the parameter t $\quad \vec{p}(s) = \vec{r}(t(s)) = \vec{r}\left(\frac{s}{\sqrt{13}}\right) = \left\langle 3\sin\left(\frac{s}{\sqrt{13}}\right), \frac{2}{\sqrt{13}}s, 3\cos\left(\frac{s}{\sqrt{13}}\right) \right\rangle$

Note: for $\vec{r}(t)$ as above so $\quad \vec{p}\,'(s) = \left\langle \frac{3}{\sqrt{13}}\cos\left(\frac{s}{\sqrt{13}}\right), \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\sin\left(\frac{s}{\sqrt{13}}\right) \right\rangle$

$$|\vec{p}\,'(s)| = \sqrt{\left(\frac{3}{\sqrt{13}}\right)^2\left(\cos^2\left(\frac{s}{\sqrt{13}}\right) + \sin^2\left(\frac{s}{\sqrt{13}}\right)\right) + \left(\frac{2}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{9}{13} + \frac{4}{13}}$$

$$|\vec{p}\,'(s)| = 1$$

So $\vec{p}(s)$ is a unit speed parameterization. (This is the general behavior)

Physicsy Nonsense ... (more or less the title of today's lecture)

Ex. Find the velocity and acceleration of

$$\vec{r}(t) = \langle e^{\ln(2)t}, t^2, \ln(t+1) \rangle \text{ at time } t=1$$

Sol. $\vec{v}(t) = \vec{r}\,'(t) = \langle \ln(2)e^{\ln(2)t}, 2t, (t+1)^{-1} \rangle$

at $t=1$ $\quad \vec{v}(1) = \langle 2\ln(2), 2, \frac{1}{2} \rangle$

$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \langle (\ln(2))^2 e^{\ln(2)t}, 2, -(t+1)^{-2} \rangle$

at $t=1$ $\quad \vec{a}(1) = \langle 2(\ln(2))^2, 2, \frac{-1}{4} \rangle$

Ex. Find velocity and position functions of the curve satisfying

$\vec{a}(t) = (2, 0, 2t)$, $\quad \vec{v}(0) = \langle 3, -1, 0 \rangle$ and $\vec{r}(0) = \langle 1, 0, 1 \rangle$

$$\vec{v}(t) = \int \vec{a}(t)\, dt = \langle 2t, 0, t^2 \rangle + \vec{c}$$

$$\langle 3, -1, 0 \rangle = \vec{v}(0) = \langle 2\cdot 0, 0, 0^2 \rangle + \vec{c}$$

$$\vec{c} = \langle 3, -1, 0 \rangle$$

$$\vec{v}(t) = \langle 2t+3, -1, t^2 \rangle$$

$$\vec{r}(t) = \int \vec{v}(t)\, dt = \langle t^2+3t, -t, \frac{t^3}{3} \rangle + \vec{d}$$

$$\langle 1, 0, 1 \rangle = \vec{r}(0) = \langle 0^2+3\cdot 0, -0, \tfrac{1}{3}0^3 \rangle + \vec{d} = \vec{d}$$

$$\vec{r}(t) = \langle t^2+3t, -t, \tfrac{1}{3}t^3 \rangle + \vec{d} = \langle t^2+3t+1, -t, \frac{t^3}{3}+1 \rangle$$

Ex. When is the particle with position function $\vec{r}(t) = \langle t^2, 5t, t^2-16t \rangle$ moving the slowest?

Sol. 1) Want to minimize speed $f(t) = |\vec{r}\,'(t)|$

$r'(t) = \langle 2t, 5, 2t-16 \rangle$ so $\qquad (2t-16)(2t-16)$

$f(t) = \sqrt{\ldots}$

$r'(t) = \langle 2t, 5, 2t-16 \rangle$ so

$f(t) = \sqrt{(2t)^2 + (5)^2 + (2t-16)^2}$

$f(t) = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$

$f(t) = \sqrt{8t^2 - 64t + 281}$

$(2t-16)(2t-16)$

$4t^2 - 64t + 256$

First derivative test

$f'(t) = \frac{1}{2}(8t^2 - 64t + 281)^{-\frac{1}{2}}(16t - 64)$

$f'(t) = \dfrac{8t - 32}{\sqrt{8t^2 - 64t + 281}}$

at $t = 4$ $f'(t) = 0$

— +

4

$f'(0) = \frac{-32}{\sqrt{281}} < 0$ $f'(5) = \frac{8}{\sqrt{\ldots}}$

The particle is slowest at time $t = 4$.

Recall: If $f(t) > 0$ for all t, then the critical points of $(f(t))^2 = g(t)$ are precisely the critical points of $f(t)$

Alt. Sol. $f(t) = \sqrt{(2t)^2 + 5^2 + (2t-16}$

so $f(t) > 5^2$ for all t

then

$g(t)$ $8t^2 - 64t + 281$

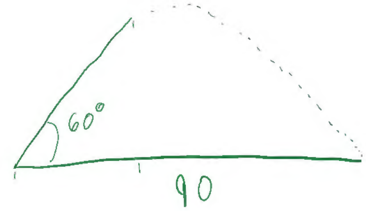
$g'(t) = 0$ iff $16t - 64 = 0$ iff $t = 4$

— +

4

Ex. A ball is kicked from the ground at an angle of $60°$. If the ball lands 90m

What was the initial speed of the ball? Gravity = 9.8 m/s²



Sol $\vec{r}(0) = \langle 0, 0 \rangle$

$\vec{a}(t) = \langle 0, -9.8 \rangle$

$\vec{V}(0) = |\vec{v}(0)| \langle \cos(\frac{\pi}{3}), \sin(\frac{\pi}{3}) \rangle = |\vec{V}(0)| \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$\vec{r}(b) = \langle 90, 0 \rangle$ Want $c = |\vec{v}(0)|$

Now $\vec{V}(t) = \int \vec{a}(t)\, dt$

$= \langle \alpha, \frac{-49}{5} t, \beta \rangle$

$c \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = \vec{V}(0) = \langle \alpha, -\frac{49}{5}, \beta \rangle$

$\vec{V}(t) = \langle \frac{1}{2} t, \frac{-49}{5} t, \frac{\sqrt{3}}{2} t \rangle$

$\vec{r}(t) = \int \vec{V}(t)\, dt = \frac{1}{2} ct + \gamma, \frac{-49}{5} t^2 + \frac{\sqrt{3}}{2} ct + \delta \rangle$

Now $\vec{r}(t) = \langle \frac{1}{2} ct, \frac{-49}{10} t^2, \frac{\sqrt{3}}{2}$

So at time b: $\begin{cases} \frac{1}{2} cb = 90 \\ -\frac{49}{10} b^2 + \frac{\sqrt{3}}{2} cb = 0 \end{cases} \rightarrow \begin{array}{l} -\frac{49}{10} b^2 + 90\sqrt{3} = 0 \\ c = \frac{180}{b} \end{array}$

$\frac{900\sqrt{3}}{49} = b^2 \Rightarrow b = \frac{30 \cdot 3^{\frac{1}{4}}}{7}$

$c = \frac{42}{3^{1/4}}$